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1984 J. Phys. A: Math. Gen. 17 L511

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J. Phys. A: Math. Gen. 17 (1984) L511–L515. Printed in Great Britain

LETTER TO THE EDITOR

Influence of long-range correlated impurities upon the phase transition in model superconductors

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Received 9 April 1984

Abstract. The influence of long-range correlated quenched impurities of a power-law decay type upon the phase transition in a model system of a superconductor is investigated. Direct renormalisation group ε-analysis yields a fixed point which is stable over a domain in a specific parameter connected with the long-range character of the correlations. The critical exponents are calculated to first order in ε. The existence of a stable fixed point is interpreted as signalling a second-order phase transition near four dimensions of space.

The purpose of this Letter is to extend the Halperin–Lubensky–Ma (HLM) investigation (Halperin *et al* 1974) by introducing long-range correlated quenched impurities of a special type (Weinrib and Halperin (WH) 1983) by means of a renormalisation-group (RG) direct perturbation ε-analysis (Wilson and Kogut 1974). The features of the same model with short-range correlations between the impurities have already been outlined by Boyanovsky and Cardy (1982) for $n=2$ and by Uzunov *et al* (1984) for any n ($n/2$ is the number of the components of the complex-order parameter field). Taking the latter work as a basis, we specify the effective Hamiltonian $\mathscr{H}=(-H/k_BT)$ of the d-dimensional model system as

$$\mathscr{H}\{A,\psi\}=-\int \mathrm{d}x[a|\psi|^2+\gamma|(\nabla-\mathrm{i}q_0A)\psi|^2+\tfrac{1}{2}b|\psi|^4+(8\pi\mu)^{-1}(\mathrm{rot}\,A)^2] \quad (1)$$

with $\psi(x)$ being the $(n/2)$-component field, $q_0=2e/\hbar c$ and the vector potential A is Coulomb-gauged: div $A(x)=0$. Now, with the intention of allowing for the influence of quenched impurities (or, equivalently, for local critical temperature fluctuations), let $a=a'(T-T_c)/T_c+\varphi(x)$. The random spatially dependent field $\varphi(x)$ is defined as a Gaussian distribution over the impurity configurations with $\langle\varphi(x)\rangle=0$, $\langle\varphi(x)\varphi(x')\rangle=g(|x-x'|)$. Consider $g(r)$, $r=|x-x'|$, as a linear superposition of inverse power law correlations decreasing with r at different rates. Then the most (and, asymptotically for $T-T_c\to 0$, the only) component of $g(r)$ relevant to the scaling analysis will be the one with the slowest decay rate at large r (WH, Kardar *et al* 1983, Chang and Abrahams 1984). So, insisting on a long-range power law, the possibility that $g(r)\sim 1/r^{\sigma}$ remains. As we use diagrammatic analysis in k-space, we need the Fourier transform of $g(r)$ which for small k is (WH)

$$\tilde{g}(k)=v+wk^{\sigma-d}. \quad (2)$$

In dealing with the influence of quenched impurities, we follow the treatment outlined by Lubensky (1975). In cases of exponential decay of $g(r)$ the $\boldsymbol{k}$-dependence of $\bar{g}(\boldsymbol{k})$ is easily ruled out of consideration on the basis of its scaling irrelevance. Now, with the power law correlation, we have one more qualitative possibility. The subtle observation of WH shows that both terms in equation (2), presumably representing two quite different (short- and long-range) types of correlations, can be 'scaling reconciled' by assuming that σ is in the vicinity of d, in the sense that $\sigma - d = O(\varepsilon)$. Here $\varepsilon = d_U - d$; d_U is the upper borderline dimensionality. As we persist in preserving the essential features of the original HLM investigation and by virtue of zero-order scaling analysis, we have $d_U = 4$, i.e. $\varepsilon = 4 - d$. Consequently, the proximity of σ and d is expressed in terms of a new parameter defined as $\delta = 4 - \sigma$, $\delta \sim O(\varepsilon)$. It embodies the long-range (LR) character of the quenched-impurity correlations.

Following the above discussion, we study the critical behaviour of the model (1) with an account for the LR correlations to first order in $\varepsilon = 4 - d$ through analysing the differential RG flow equations (Wegner and Houghton 1973):

$$\mathrm{d}\gamma/\mathrm{d}\rho = -\eta\gamma - (3/2\pi)t, \qquad \mathrm{d}\mu/\mathrm{d}\rho = \eta_A\mu - (n/12\pi)t, \tag{3a, b}$$

$$\mathrm{d}(q_0^2)/\mathrm{d}\rho = -\eta_A + \varepsilon, \tag{3c}$$

$$\mathrm{d}a/\mathrm{d}\rho = (2-\eta)a + \tfrac{1}{2}[Ky/(1+a)]\cdot(n+2)/b - [K_y/(1+a)(v+w) + (3/2\pi)\gamma t, \tag{3d}$$

$$\mathrm{d}b/\mathrm{d}\rho = (\varepsilon - 2\eta)b - \tfrac{1}{2}K_y(n+8)b^2 + 6K_y b(v+w) - 12t^2\gamma^2, \tag{3e}$$

$$\mathrm{d}v/\mathrm{d}\rho = (\varepsilon - 2\eta)v - K_y(n+2)bv + 2K_y v(v+w) + 2K_y(v+w)^2 \tag{3f}$$

and

$$\mathrm{d}w/\mathrm{d}\rho = (\delta - 2\eta)w - K_y(n+2)bw + 2K_y w(v+w). \tag{3g}$$

Here η and η_A are the anomalous dimensionalities of the order parameter and the vector potential respectively, $t = \mu q_0^2$, $K_d^{-1} = 2^{d-1}\pi^{d/2}\Gamma(d/2)$ ($\Gamma(z)$ is the gamma function). We have the momentum cut-off $\Lambda = 1$; also, we set $a = 0$ in equations (3e, f, g) which is correct to $O(\varepsilon)$ in view of $a^* \sim O(\varepsilon)$. The $\boldsymbol{k}$-dependence of the impurity lines (cf (2)) does not introduce new terms proportional to k^2 in the two-point order parameter vertex. Under the usual invariance condition on γ, we obtain $\eta = -18\epsilon/n$ and $\eta_A = \varepsilon$ when the fixed point (FP) value of t is $t^* = (12\pi/n)\varepsilon$ (the other possibility, $t^* = 0$, is trivial if we are interested in the influence of the vector potential). The values of η and η_A are the same as in the pure and the short range (SR) case.

We make the natural change of variables: $r = a/\gamma$, $u = b/8\pi^2\gamma^2$, $\Delta_1 = v/2\pi^2\gamma^2$, $\Delta_2 = w/2\pi^2\gamma^2$ and define $\Sigma = \Delta_1 + \Delta_2$ and $y = \varepsilon/\delta$. In searching for FPs of the flow equations we do not consider unphysical FPs (Lubensky (1975); for a detailed discussion on the allowed values of u, v and w and, hence, of their FP values, see WH). Also, we do not present the FP value of r as it is not necessary for the calculation of the critical exponents to first order in ε.

Our main interest lies in finding a LR FP, i.e. $\Delta_2^* \neq 0$. Equations (3) do have such a FP:

$$u_\pm^* = \frac{\delta}{n(5n+y)}(n(3-y) + 72y \pm \{[n(3-y)+72y]^2 + 432(5n+4)y^2\}^{1/2}), \tag{4a}$$

$$\Sigma_\pm^* = 2(n+2)u_\pm^* - 2\delta(1 + (36/n)y), \tag{4b}$$

$$\Delta_{1,\pm}^* = (\Sigma_\pm^*)^2/2\delta(1-y), \tag{4c}$$

$$\Delta_{2,\pm}^* = \Sigma_\pm^* - (\Sigma_\pm^*)^2/2\delta(1-y). \tag{4d}$$

The immediate observation is that one more parameter (δ) is involved in the FP values in comparison with the SR case (Boyanovsky and Cardy 1982, Uzunov *et al* 1984). Already this brings about a complication at the level of the sign of δ as we cannot exclude the possibility of both $u^*_\pm$ being positive as required by stability of the Hamiltonian. Namely, the expression in the parentheses of equation (4*a*) is positive for the upper sign and negative for the lower sign independently of n and y, hence, $u^*_+ > 0$ for $\delta > 0$ and $u^*_- > 0$ for $\delta < 0$. So, to make the discussion more lucid, we deal with the two LR possibilities separately.

First, consider $\delta > 0$ and $u^*_+ > 0$ which turns out to be the more interesting case in discussing the LR critical behaviour. When $\varepsilon > \delta > 0$, one cannot meet the condition $\Delta^*_{1,+} \geq 0$ (WH). Therefore, $\delta > \varepsilon$ and $\delta > 0$, so $|y| < 1$. Rewriting the flow equations (3) in the new variables and linearising them around the FP (4) taken with the upper sign, we obtain

$$\nu = 2/\sigma = \tfrac{1}{2} + \tfrac{1}{8}\delta + O(\epsilon^2). \tag{5}$$

Contrary to the SR case (Uzunov *et al* 1984), the value of ν is decoupled even from its indirect connection with the presence of the magnetic field (through the value of u^*_+). In fact, ν coincides with that of WH and as their arguments remain unaltered here $\nu = 2/\sigma$ might be regarded as the exact value of ν. Equation (5) provides evidence of the strong influence of the LR correlated impurities on the critical behaviour of the model described by equation (1).

At this point we fix $n = 2$ to study further the particular case of a superconductor. To investigate the stability of the LR FP (4), we solve numerically the third-degree characteristic equation for the eigenvalues μ_i, $i = 1, 2, 3$, connected with the irrelevant parameters u, Δ_1 and Δ_2. The FP is stable when the real parts of all the three critical exponents μ_i are negative. In the present problem this turns out to be so in the domain of values for $y = \varepsilon/\delta$ between $y_L = -0.016$ and $y_R = 0.054$. It is worth noting that the domain of stability $\Delta y = y_R - y_L$ is smaller than the respective one in WH for $n = 2$ by an order of magnitude. For $y_L < y < y_R$ we have one real and a pair of complex conjugate irrelevant critical exponents. Therefore, here the LR FP (4) is of the focus type leading to oscillating corrections to scaling (Khmel'nitskii 1978). The qualitative behaviour of the exponents μ_i (measured in units δ) on the interval $[y_L, y_R]$ is as follows: the real part of the conjugate pair $\mu_{1,2}$ decreases monotonously from zero at y_L to (-0.770) at y_R (the corresponding imaginary parts being 0.934 and 1.093); the real eigenvalue μ_3 equals (-0.633) at y_L and is zero at y_R reaching its minimal value of (-0.751) on the interval under consideration at $y = 0.007$. The implications of the zeros for the crossover behaviour are discussed below.

It is obvious that the flow equations (3) have a FP with $\Delta^*_2 = 0$ (a short-range FP). The nice thing about it is that we recover the expected exchange of stability (crossover) of the LR and the SR FPs at the upper critical value $y_R = \varepsilon/\delta = 0.054$ ($n = 2$). At $n = 2$ the other possible FP of equations (3) corresponding to the pure (HLM) case is unstable.

Let us go beyond the restriction $n = 2$ in clarifying the crossover behaviour (note that at an earlier stage we imposed the condition $t^* \neq 0$ and we are still at $\delta > 0$). Below $n = 366$ the crossover is between LR and SR critical behaviour. In the region $n \in [2, 366]$, the upper critical value y_R increases to reach $y_R = 0.999$ at $n = 366$ whereupon $y_L = -0.27$ (for $y_R > 1$ the LR FP becomes negative, i.e. becomes unphysical). The exact crossover value y_R can be obtained from stability analysis of the long- and short-range FPs for any given value of n or, equivalently, via the extended Harris criterion (WH) with the help of the result for $\nu_{\text{short}}(\varepsilon, n)$ from the paper of Uzunov *et* *al* (1984). The same observation holds for the crossover from LR to the pure case that takes place for $n > 366$ with ν_{pure} from HLM. In this case the specific dependence of $\nu(n)$ leads to a decrease in the upper critical value y_R back from $y_R \approx 1$ (for instance, at $n = 500$, $y_R = 0.666$). Note that, not surprisingly, the marginal value of n (that is, the one at which the crossover 'neighbours' of the LR stable critical behaviour change) lies between 366 and 367 and, strictly speaking, is non-integer.

Another interesting peculiarity is that for $n \geqslant 366$ a subdomain of the region $[y_L, y_R]$ develops comprising the lower vicinity of y_R in which the exponents μ_i are all real (and negative). However small this subdomain is (for instance, at $n = 500$ it is $[0.059, y_R = 0.666]$), its existence indicates a normal and not a focus-type stability of the FP (4) with u^*_+. Besides, the size of the subdomain also increases with n. Note that the discussion on the stability of the LR FP (4) is independent of whether we have a focus-type or a normal FP; we concentrated only upon the negative signs of the real parts of the exponents.

Crossing the lower critical value y_L of the domain of stability, the LR FP remains positive, but loses stability because the real part of the conjugate pair of irrelevant exponents goes positive inducing a spiral runaway behaviour. The instability sets in at y_L through a subcritical Hopf bifurcation as has been checked explicitly for the case $n = 2$.

Now let us consider briefly the case $\delta < 0$ and $u^*_\pm > 0$ (equations (4)) which is the second possibility for a LR FP. Here $\delta < \varepsilon$ cannot meet the requirement $\Delta^*_{1,-} > 0$. So $0 > \delta > \varepsilon$ and $y > 1$. With the due restrictions $\Delta^*_{2,-} > 0$ and $\Sigma^*_\pm > 0$ (WH) the numerical analysis yields a physical LR FP in the region of the (n, y)-plane specified by $n \geqslant 193$ (n integer) and $y \geqslant y_n$; y_n is ≈ 1295 at $n = 193$ and decreases to approach 2 with the increase of n. However, it is unstable towards perturbations in all the irrelevant parameters as the irrelevant exponents have positive real parts throughout the indicated region.

With all that information in mind we may try to draw a consistent conclusion as to the order of the phase transition signalled by the existence of the stable LR FP. In the spirit of the RG interpretation of critical behaviour we have to conclude that the system exhibits a second-order phase transition in the domain of stability of the FP indicated explicitly for the case $n = 2$. However, we cannot extend this conclusion to $d = 3$. A simple scaling consistency condition (non-negativity of the set of relevant scaling exponents) imposes once again (cf Uzunov *et al* 1984) the restriction $|\varepsilon| < 0.2$ at $n = 2$, that is, we do not have even the virtual possibility of speculating of $\varepsilon = 1$ and, consequently, of $d = 3$. So, at $d = 3$, one must rely on the original argument of HLM for a fluctuation-induced first-order phase transition that can be extended to the quenched impurity case without alterations. The recent considerable development towards accounting for the fluctuations in the order parameter in reasoning the order and the 'size' of the transition proved that this is not straightforward even in the pure case (Lawrie 1983). There is little hope that the introduction of LR correlated impurities of the type discussed here would lift the ambiguity of the persistence of an arbitrary magnetic field (Lawrie 1983), because the power law correlation has no characteristic scale and, besides, the choice of the decay rate is strongly bound by the $\delta = O(\varepsilon)$ ansatz.

We acknowledge with thanks the critical and stimulating discussions with Dr D I Uzunov at all stages of the investigation. We also thank K Korutchev for helpful instructions on the numerical analysis.

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